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EVALUATING THE RESIDUAL STRENGTH OF SHELLS OF LAMINATED COMPOSITES WITH THROUGH-SLIT-TYPE DEFECTS

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Composite shells are being used more and more in different areas of technology (for pressure vessels, curved panels of aircraft skins, etc.). An analysis of the experimental studies and past use of structures made of polymer composites (plastics reinforced with fibers of glass, carbon, or boron) shows that the damage incurred by such structures is due to not to the working loads but to random mechanical effects (shocks imparted during assembly and adjustment, collisions with stones and hailstones, etc.). Moreover, damage increases very slowly over the range of working loads [1, 2]. Thus, in contrast to metallic elements, the service life of polymeric composite structures is determined mainly by their residual strength. Analytical and experimental studies have covered a wide range of problems concerning the failure of shells with crack-like defects (see the survey in [3], for example), but the investigation of this problem for composite materials is progressing relatively slowly, due to a shortage of information on the effect on crack resistance of such characteristics as the relative dimensions of the shell and crack, the order in which the layers are arranged, and the type of anisotropy of the monolayers.

Here, we use the method of integral boundary equations and a strain criterion of the strength of laminated polymer composites to propose a closed algorithm for calculating the stress state near a defect. We will use the same approach to evaluate residual strength for a shell with a system of through slits. We present numerical results illustrating the effect of different geometric and physical parameters of the problem on the relative stress intensity factors and critical load.

1. We will examine an infinite cylindrical shell of radius R and constant thickness 2h composed of uniform anisotropic layers arranged symmetrically relative to the middle surface. The shell has a system of through slits of length $2l_0$ ($l_0 \gg h$) directed at an angle ϑ to the coordinate line α_2 of local orthogonal dimensionless coordinates $\alpha_1 O \alpha_2$, referred to the radius of the shell R and connected with its middle surface (Fig. 1). The parametric equations of the slits have the form $\alpha_1 = \xi_1(\lambda) = -n_2\lambda$, $\alpha_2 = \xi_2(\lambda) = n_1\lambda$, $|\lambda| \le \lambda_0 = R^{-1}l_0 + mT$ ($m = 0, 1, \ldots, \mu - 1$), where $\nu = (n, n_2) = (\cos\vartheta, \sin\vartheta)$ is a unit normal to the left edge of the slit; $T = 2\pi/\mu$ is the period; μ is the number of slits.

Proceeding on the basis of relations from the theory of shallow anisotropic shells with internal stresses [3, 4], we represent the components of the tensor of the small strains in the form

$$e_{ij} = e_{ij}^{e} + e_{ij}^{0} \quad (i, j = 1, 2).$$

Here, e_{ij} are components of the total-strain tensor; e_{ij}^{0} are components of the strain tensor for the material free of stresses; e_{ij}^{e} are components of the tensor of the elastic strains caused by the internal stresses σ_{ij}. These strains and stresses are connected by the generalized Hooke's law for anisotropic bodies.

We now replace the internal stresses σ_{ij} by their statically equivalent forces $N = \{N_1, S, N_2\}^T$ and moments $M = \{M_1, H, M_2\}^T$, and instead of components of the small-strain tensor e_{ij} we use components of the strain of the middle surface $\varepsilon = \{\varepsilon_{11}, \varepsilon_{12}, \varepsilon_{22}\}^T$, $\varkappa = \{\varkappa_{11}, 2\varkappa_{12}, \varkappa_{22}\}^T$ (where T denotes transposition). These strains are in turn connected by known relations with the displacements of the middle surface $u_i = u_i(\alpha_1, \alpha_2)$ ($i = 1, 2$), $w = w(\alpha_1, \alpha_2)$ and angles of rotation $\theta_i = \theta_i(\alpha_1, \alpha_2) = \partial_i w/R$, $\partial_i = \partial/\partial\alpha_i$ ($i = 1, 2$) [3-5].

Using Hooke's law and the above assumptions, we represent the relationship between the forces, moments, and strains of the middle surface in matrix form [4, 5]

$$N = C\varepsilon^{e}, \quad M = D\varkappa^{e}; \tag{1.1}$$

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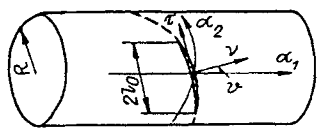

Fig. 1

$$\varepsilon_{ij}^{e} = \varepsilon_{ij} - \varepsilon_{ij}^{0}, \quad \varkappa_{ij}^{e} = \varkappa_{ij} - \varkappa_{ij}^{0},$$
$$\varepsilon_{ij}^{0} = \frac{1}{2h}\int_{-h}^{h} e_{ij}^{0} d\gamma, \quad \varkappa_{ii}^{0} = \frac{3}{2h}\int_{-h}^{h} e_{ii}^{0}\gamma d\gamma, \tag{1.2}$$
$$2\varkappa_{12}^{0} = \frac{3}{2h}\int_{-h}^{h} e_{12}^{0}\gamma d\gamma,$$

where **C** and **D** are the stiffness matrices of the shell. The components of these matrices are connected with the elastic characteristics of the layers.

Without allowance for the surface load, the equilibrium equation of a shell with internal stresses can be written in operator form [4]

$$\mathrm{F}_N \mathrm{N} + R^{-1}\mathrm{F}_M \mathrm{M} = 0; \tag{1.3}$$

$$\mathrm{F}_N = \begin{bmatrix} \partial_1 & \partial_2 & 0 \\ 0 & \partial_1 & \partial_2 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathrm{F}_M = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\partial_1^2 & -2\partial_1\partial_2 & -\partial_2^2 \end{bmatrix}. \tag{1.4}$$

Introducing the notation $\mathbf{u} = \{u_1, u_2, w\}^T$, we rewrite (1.1) as follows with allowance for (1.2) and (1.4):

$$\mathrm{N} = R^{-1}\mathrm{G}_N \mathrm{u} - \mathrm{C}\boldsymbol{\varepsilon}^0, \quad \mathrm{M} = R^{-2}\mathrm{G}_M \mathrm{u} - \mathrm{D}\boldsymbol{\varkappa}^0,$$
$$\mathrm{G}_N = \mathrm{C}\mathrm{F}_N^{\mathrm{T}}, \quad \mathrm{G}_M = \mathrm{D}\mathrm{F}_M^{\mathrm{T}}. \tag{1.5}$$

Inserting (1.5) into (1.3), we use the theory of shallow anisotropic shells with internal stresses to obtain a system of equations in displacements:

$$\mathrm{L}\mathrm{u} = R\mathrm{G}_N^{\mathrm{T}}\boldsymbol{\varepsilon}^0 + \mathrm{G}_M^{\mathrm{T}}\boldsymbol{\varkappa}^0. \tag{1.6}$$

Here, $\mathbf{L} = \mathbf{F_N C F_N}^T + \mathbf{R}^{-2}\mathbf{F_M D F_M}^T$ is a symmetric differential matrix operator. The particular solution of system (1.6) can be written in the form

$$\mathrm{u} = \mathrm{L}_* (R\mathrm{G}_N^{\mathrm{T}}\boldsymbol{\varphi} + \mathrm{G}_M^{\mathrm{T}}\boldsymbol{\psi}), \tag{1.7}$$

where $\mathbf{L}_*$ is a differential matrix operator whose elements L_{*ij} are algebraic complements of elements L_{ij} of the symmetric matrix **L**, while the vector functions $\psi = \{\varphi_1, \varphi_2, \varphi_3\}^T$ and $\varphi = \{\psi_1, \psi_2, \psi_3\}^T$ satisfy the equations

$$\Lambda\boldsymbol{\varphi} = \boldsymbol{\varepsilon}^0, \quad \Lambda\boldsymbol{\psi} = \boldsymbol{\varkappa}^0, \quad \Lambda = \det \|\mathrm{L}\|. \tag{1.8}$$

We write the eighth-order differential operator Λ in the form

$$\Lambda = R^{-2}\Omega\Delta, \quad \Delta = \Delta^8 + R^2\partial_1, \quad \Delta^8 = R_*^4 Q_*^4, \quad \Omega = \det \|\mathrm{C}\|. \tag{1.9}$$

Here, $R_*^4 = D_{11}\partial_1^4 + 4D_{16}\partial_1^3\partial_2 + 2(D_{12} + 2D_{66})\,\partial_1^2\partial_2^2 + 4D_{26}\partial_1\partial_2^3 + D_{22}\partial_2^4$; $Q_*^4 = A_{22}\partial_1^4 - 2A_{26}\partial_1^3\partial_2 + (A_{66} + 2A_{12})\cdot \partial_1^2\partial_2^2 - 2A_{16}\partial_1\partial_2^3 + A_{11}\partial^4$; D_{ij} (i, j = 1, 2, 6) are elements of the matrix **D**; A_{ij} (i, j = 1, 2, 6) are elements of the matrix which is the inverse of **C** [5].

It is evident that the stress state in a shell with a system of slits will be of a T-periodic character. Without loss of generality, we will restrict further discussion to the case of a single slit: $\alpha_1 = -n_2\lambda$, $\alpha_2 = n_1\lambda$, $|\lambda| \leq \lambda_0$.

The solution of the equilibrium equation must satisfy the value assigned along the contour of the slit for the boundary vector of the generalized (in the Kirchhoff sense) forces and moments [4]

$$\mathrm{T} = \mathrm{VN} + R^{-1}\mathrm{WM}; \tag{1.10}$$

$$\mathrm{V} = \begin{bmatrix} n_1^2 & 2n_1n_2 & n_2^2 \\ -n_1n_2 & n_1^2 - n_2^2 & n_1n_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\mathrm{W} = \begin{bmatrix} -n_1^2n_2^2 & 2n_1^3n_2 & n_1^2n_2^2 \\ -n_1^3n_2 & 2n_1^4 & n_1^3n_2 \\ n_1^2 & 2n_1n_2 & n_2^2 \\ n_1^2\partial_1' + 2n_1n_2\partial_2' & 2\,(n_1n_2\partial_1' + (n_1^2 - n_2^2)\,\partial_2') & n_2^2\partial_1' + 2n_1n_2\partial_2' \end{bmatrix} \tag{1.11}$$

$$(\partial_1' = n_1\partial_1 + n_2\partial_2, \quad \partial_2' = -n_2\partial_1 + n_1\partial_2).$$

2. We will examine a problem concerning the equilibrium of a cylindrical shell under a specified external load. Equal but oppositely directed components of the boundary vector $\mathbf{T}^{\pm} = \mathbf{T}(\xi_1(\lambda) \pm n_1 0,\ \xi(\lambda) \pm n_2 0)$, $|\lambda| \leq \lambda_0 = R^{-1}l_0$ are applied to the edges of the slit. We will represent the stress state in the shell as the sum of the main stress state due to the external load and the perturbed stress state due to the presence of the slit. We use $\mathbf{N}^1$ and $\mathbf{M}^1$ to represent the vectors of the generalized forces and moments which arise in the shell with a slit due to the external load under assigned boundary conditions (on the slit), while $\mathbf{N}^2$ and $\mathbf{M}^2$ will denote the forces and moments in the shell in the absence of a slit (but with the same external load). Due to the linearity of the problem, we can represent the forces and moments in the shell with the slit as $\mathbf{N}^1 = \mathbf{N}^2 + \mathbf{N}$, $\mathbf{M}^1 = \mathbf{M}^2 + \mathbf{M}$, where $\mathbf{N}$ and $\mathbf{M}$ are the vectors of the generalized forces and moments due to the presence of the slit (the perturbed stress state [3]). We assume that the main stress state is known. Furthermore, we will limit ourselves to consideration of those cases in which the edges of the slit do not come into contact with one another due to deformation.

Thus, the perturbed stress state on the line of the slit should satisfy the condition

$$\mathrm{T}^+ = \mathrm{T}^- = \mathrm{f}\,(\xi_1(\lambda),\ \xi_2(\lambda)), \quad |\lambda| \leqslant \lambda_0 = R^{-1}l_0, \tag{2.1}$$

where $\mathbf{f} = \mathbf{T}^1 - \mathbf{T}^2$. The signs + and − denote boundary values on the left and right edges of the slit.

In accordance with the above description of a shell with a slit, we assume the existence of an identical, solid shell with internal stress sources (dislocations and disclinations) concentrated on the line of the slit. Here, we require that the densities of these sources be distributed along the slit line in such a way that the stress state coincide with the stress state in the shell with the slit. In accordance with (2.1), the forces and moments remain continuous at an arbitrary point of the shell, while the functions describing the displacements u_1, u_2, and w and the angles of rotation θ_1, θ_2 undergo first-order discontinuities in the transition across the slit line. Differentiating them as generalized functions on the basis of (1.2), we express the components of the strains ε_{ij}^0, $\varkappa_{ij}^0$ through functionals concentrated on the slit line. The densities of the latter are combinations of the jumps of the displacements and angles of rotation [3]:

$$\begin{gathered} \varepsilon_{ii}^0 = R^{-1}\,(n_i\,[u_i],\ \delta)_L \quad (i = 1,\ 2), \\ \varepsilon_{12}^0 = R^{-1}\,((n_2\,[u_1] + n_1\,[u_2]),\ \delta)_L, \\ \varkappa_{ii}^0 = -R^{-1}\,((n_i\,[\theta_i],\ \delta)_L + R^{-1}\,(n_i\,[w],\ \partial_i\delta)_L) \quad (i = 1,\ 2), \\ \varkappa_{12}^0 = -R^{-2}\,((n_1\partial_1 + n_2\partial_2)\,[w],\ \delta)_L. \end{gathered} \tag{2.2}$$

Here, $\delta = \delta(\alpha_1, \alpha_2) = \delta(\alpha_1)\,\delta(\alpha_2)$ is the delta function; $[f] = f^+ - f^-$ is the jump of the function f associated with the transition across the line of the slit; $(f, g)_L = \int f(\xi_1(\lambda), \xi_2(\lambda))\, g(\alpha_1 - \xi_1(\lambda), \alpha_2 - \xi_2(\lambda))d\lambda$ is the convolution integral of the functions f and g along the line of the slit $|\lambda| \leq \lambda_0$. Thus, the strains ε_{ij}^0, $\varkappa_{ij}^0$ (i, j = 1, 2) correspond to concentrated factors distributed along the slit line in the solid shell. The densities of these factors are combinations of the jumps of the displacements and angles of rotation.

We will use the fundamental solution $E(\alpha_1, \alpha_2)$ of the eighth-order operator Δ from (1.9) [5] and, by virtue of (1.6)-(1.9), write the resolvent functions $\varphi(\alpha_1, \alpha_2)$, $\psi(\alpha_1, \alpha_2)$ in the form

$$\varphi = R^2\Omega^{-1}(\varepsilon^0, E)_L, \quad \psi = R\Omega^{-1}(\varkappa^0, E)_L$$

or, with allowance for (1.11), (2.2), in the form

$$\varphi = R^2\Omega^{-1}(\Phi, V^{\mathrm{T}}E)_L, \quad \psi = R\Omega^{-1}(\Phi, W^{\mathrm{T}}E)_L, \tag{2.3}$$

where $\Phi = \{R^{-1}[u_\nu], R^{-1}[u_\tau], -[\theta_\nu], -R^{-1}[w]\}^T$ is the vector of the jumps of the generalized (in the Kirchhoff sense) displacements and angles of rotation with the transition across the slit line (the subscripts ν and τ denote the normal and tangential components).

Using relations (1.5), (1.8), and (2.3), we obtain an integral representation of the vector of the generalized forces and moments for the perturbed state of the shell:

$$T = (\Phi, T_* E)_L. \tag{2.4}$$

Here, $T_* = [T_{*ij}]$ (i, j = 1, ..., 4) is a symmetric differential matrix operator with eighth-order partial derivatives:

$$\begin{gathered}
T_* = \Omega^{-1}(R^2 V N_\varphi V^{\mathrm{T}} + V N_\psi W^{\mathrm{T}} + W M_\varphi V^{\mathrm{T}} + W M_\psi W^{\mathrm{T}}),\\
N_\varphi = G_N L_* G_N^{\mathrm{T}} - C\Lambda, \quad N_\psi = G_N L_* G_M^{\mathrm{T}},\\
M_\psi = R^{-2} G_M L_* G_M^{\mathrm{T}} - D\Lambda, \quad M_\varphi = G_M L_* G_N^{\mathrm{T}}.
\end{gathered}$$

3. We insert (2.4) into boundary condition (2.1) and, reducing the order of the partial derivatives of the operator T_* so that the integral representation contains no derivatives higher than the seventh order, we construct a system of four singular integral equations to determine four unknown functions φ_i (i = 1, ..., 4):

$$\sum_{i=1}^{4}\int_{-1}^{1}\varphi_i(x)\,K_{ij}(x-y)\,dx = F_j(y) \quad (j = \overline{1, 4}). \tag{3.1}$$

Here, $\varphi_m(z) = \Phi_m'(\lambda)$; $\varphi_4(z) = \Phi_4''(\lambda)$; $F_m(z) = f_m(\lambda)$; $F_4(z) = \int f_4(\lambda)d\lambda + C_0$ (m = 1, ..., 3); $\lambda = \lambda_0 z$ ($|z| \leq 1$); C_0 is the constant of integration.

The solution of system (3.1) should satisfy the additional limitations [3]

$$\int_{-1}^{1}\varphi_i(x)\,dx = 0 \quad (i = \overline{1, 4}), \quad \int_{-1}^{1}x\varphi_4(x)\,dx = 0, \tag{3.2}$$

which serve to determine C_0 and ensure that the displacements and angles of rotation at the vertices of the slit are unambiguous.

The kernels K_{ij} (i, j = 1, ..., 4) have a Cauchy-type singularity, and the singular integrals in (3.1) are taken as the main Cauchy values. The behavior of the solutions in the neighborhood of the singular points (the tips of the slit) is known [3, 6], and the sought functions $\varphi_i(x)$ (i = 1, ..., 4) can be represented in the form

$$\varphi_i(x) = (1-x)^{-1/2}\varphi_i^0(x) \quad (i = \overline{1, 4}),$$

where $\varphi_i^0(x)$ is a new unknown function. This function is continuous and is bounded on the segment $|x| \leq 1$.

We will construct an approximate solution of system (3.1), (3.2) by the method of mechanical quadratures. This method allows us to reduce the initial system of singular integral equations to a system of linear algebraic equations for the determination of new unknown functions $\varphi_i^0(x_m)$ (i = 1, ..., 4) at specified division points of the interval of integration $x_m = \cos((2m-1) \times \pi/2M)$ (m = 1, ..., M). These functions are the roots of a Chebyshev polynomial of the first kind $T_M(x) = \cos(M \arccos(x))$ [7]:

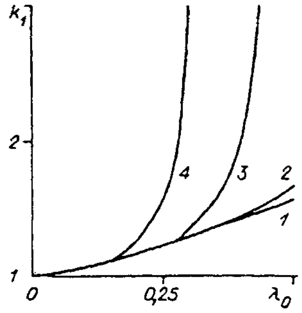

Fig. 2

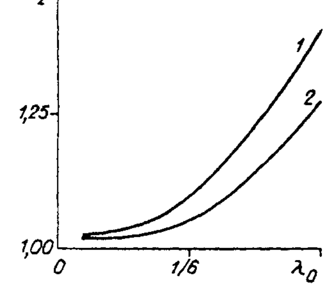

Fig. 3

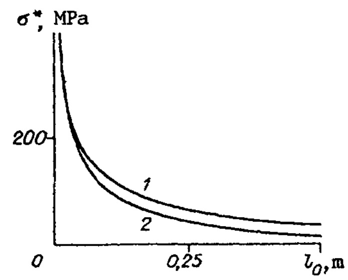

Fig. 4

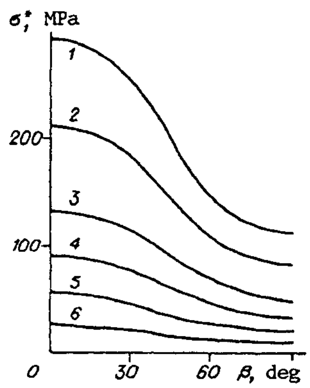

Fig. 5

$$\sum_{m=1}^{M}\sum_{i=1}^{4}\varphi_i^0(x_m)\,K_{ij}(x_m-y_r)=(M/\pi)\,F_j(y_r)\quad(r=\overline{1,M-1}),$$

$$\sum_{m=1}^{M}\varphi_i^0(x_m)=0\quad(i=\overline{1,4}),\quad\sum_{m=1}^{M}x_m\varphi_4^0(x_m)=0.$$

Here, $y_r=\cos(r\pi/M)$ ($r=1, M-1$) are roots of a Chebyshev polynomial of the second kind $U_{M-1}=\sin(M\arccos(x))(1-x^2)^{-1/2}$.

The limiting values $\varphi_i^0(\pm1)$ ($i = 1, \ldots, 4$) are calculated by using the Lagrangian interpolation formulas at the Chebyshev nodes:

$$\varphi_i^0(1)=(1/M)\sum_{m=1}^{M}(-1)^{m+1}\varphi_i^0(x_m)\,\mathrm{ctg}\,((2m-1)\pi/4M),$$

$$\varphi_i^0(-1)=(1/M)\sum_{m=1}^{M}(-1)^{m+M}\varphi_i^0(x_m)\,\mathrm{tg}\,((2m-1)\pi/4M).$$

As characteristics of the stress-strain state of the shell with the slit, we examine the stress-intensity factors [3]:

$$K_i=R^{\lambda_i^3}\lim_{\rho\to0}\left(l_0^{\lambda_i^4}(2\rho/l_0)^{1/2}T_i(\lambda_0+R^{-1}\rho)\right)\quad(i=\overline{1,4})$$

($T_i(\lambda_0+R^{-1}\rho)$ ($i = 1, \ldots, 4$) are components of the vector of the generalized forces and moments (1.10) on the continuation of the slit line). With the retention of only the principal terms in (2.4), we can determine their values near the tips of the slit

TABLE 1

M	k_1	σ^*, MPa	Computing time, min
5	1,5454179694	37,5390134387	5
9	1,5821835276	36,6673077758	15
15	1,5822903898	36,6648330731	45

by replacing the Green's function $E(\alpha_1, \alpha_2)$ by its principal value $E_*(\alpha_1, \alpha_2)$: $\Delta^8 E_*(\alpha_1, \alpha_2) = \delta(\alpha_1, \alpha_2)$ (Δ^8 is the differential operator from (1.9)).

4. Shown below are results of calculations performed for a cylindrical shell weakened by a circular slit (the normal to the contour of the slit is directed along the line α_1). The edges of the slit are free of stresses, while either a constant tensile force $N_1^\infty = P$ or a constant shearing force $S^\infty = P$ is assigned at infinity. The edges of the slit do not come into contact during deformation, so we checked for satisfaction of the condition $[u_1(\alpha_2)] \pm h[\theta_1(\alpha_2)] > 0$, $|\alpha_2| < \lambda_0$ when we obtained the solution. The calculations were performed for a shell composed of orthotropic monolayers having the elastic characteristics $E_{11} = 145$ GPa, $E_{22} = 9.5$ GPa, $G_{12} = 5.2$ GPa, $\nu_{12} = 0.31$ (material 1); $E_{11} = 255$ GPa, $E_{22} = 159$ GPa, $G_{12} = 50$ GPa, $\nu_{12} = 0.23$ (material 2).

Figure 2 shows graphs of the relative stress-intensity factor $k_1 = K_1/P$ for an orthotropic shell of material 1 subjected to a tensile force $N_1^\infty = P$. The radius of the shell $R = 1$ m, while the thickness $h = 0.01$ m. The shell has a periodic system $\mu = 1; 5; 7; 10$ of circular slits (curves 1-4, respectively). As the number of slits increases, their interaction has a substantial effect on k_1.

Figure 3 shows graphs of the relative stress-intensity factor $k_2 = K_2/P$ in an orthotropic shell of material 2 subjected to a shearing force $S^\infty = P$. The calculations were performed for shells with a relative thickness $h/R = 1/60; 1/30$ (curves 1 and 2). It is evident that the value of k_2 depends significantly both on the parameter λ_0 and on the relative thickness of the shell.

Using the above approach to study the stress-strain state of anisotropic laminated shells with a slit, we can analyze the way in which the residual strength of the shell is affected by such parameters of the problem as the type of external load, the dimensions of the shell and slit, and the order in which the layers are arranged. As an illustration of the effect of slit length and shell thickness on residual strength, we present the results of calculations performed for an orthotropic shell of material 1 with a circular slit. The shell was acted upon by the tensile force $N_1^\infty = P$. The calculations were performed using the strain criterion in [8]. The validity of this criterion has been demonstrated experimentally for a wide range of laminated composites with different arrangements of layers. Figure 4 shows graphs of the critical load $\sigma^* = N_1^*/2h$. The principal direction of orthotropy coincides with the α_1 direction. The radius of the shell $R = 1$ m, while its thickness $2h = 0.02; 0.002$ m (curves 1 and 2). An analysis of the graphs shows that the critical load decreases markedly with an increase in the length of the slit and, for large slits, also depends on the thickness of the shell.

Figure 5 shows results of calculations of the critical load σ^* of a laminated shell with a circular slit. The shell was subjected to a constant tensile force $N_1^\infty = P$. The calculations were performed for different orientations $[0; \pm\beta; 90°]_s$ of monolayers of the same thickness (the numbers in the brackets denote the angles between the principal directions of orthotropy of the monolayers and the angle α_1; the subscript s denotes the symmetry in the location of the monolayers relative to the middle surface). The elastic characteristics of the monolayers correspond to material 1. The radius of the shell $R = 1$ m, thickness $2h = 0.02$ m, slit length $l_0 = 0.01; 0.02; 0.05; 0.1; 0.2; 0.5$ m (curves 1-6). The results of the calculations show that an increase in β and slit length is accompanied by a substantial decrease in the critical load.

In the problems being examined, the singular integral equations can be solved with satisfactory accuracy by a relatively coarse subdivision of the interval of integration. All of our calculations were performed for $M = 5$ Chebyshev nodes. When necessary, the number of points of integration was increased to $M = 9$. For $M = 5, 9, 15$, Table 1 shows the results of calculations performed for an orthotropic shell of material 1 subjected to the tensile force $N_1^\infty = P$ ($R = 1$ m, $h = 0.01$ m, $l_0 = 0.5$ m). It is evident that an increase in the number of points from $M = 5$ to $M = 9$ improves the accuracy of the results 2-3%, while computing time increases threefold. At $M = 15$, computing time increases by the same amount, but the results are refined only to the fifth significant digit.

The examples presented above allow us to conclude that the proposed method of calculating the stress state and evaluating the residual strength of polymeric composite shells with slits is both simple and effective.

REFERENCES

1. A. F. Selikhov, V. F. Kut'inov, and A. E. Ushakov, "Aspects of ensuring the strength and safety of aircraft structures made of polymer composites with allowance for their damage. 1. Basic principles," Mekh. Kompozitn. Mater., No. 3 (1990).
2. K. Y. Cang, Liu Sheng, and F. K. Chang, "Damage tolerance of laminated composites containing an open hole and subjected to tensile loads," J. Compos. Mater., **25**, No. 3 (1991).
3. V. A. Osadchuk, Stress–Strain State and Limit Equilibrium of Shells with Slits [in Russian], Naukova Dumka, Kiev (1985).
4. S. A. Ambartsumyan, Theory of Anisotropic Shells [in Russian], Fizmatgiz, Moscow (1961).
5. V. M. Maksimenko and L. A. Fil'shtinskii, "Elastic behavior of anisotropic shells under loads concentrated on lines," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 5 (1974).
6. N. I. Muskhelishvili, Singular Integral Equations [in Russian], Fizmatgiz, Moscow (1962).
7. S. M. Belotserkovskii and I. K. Lifanov, Numerical Methods in Singular Integral Equations [in Russian], Nauka, Moscow (1985).
8. C. C. Poe, "A unifying strain criterion for fracture of fibrous composite laminates," Eng. Fract. Mech., **17**, No. 2 (1983).